Test 3 - MTH 1420

Dr. Graham-Squire, Spring 2012

Name: _____

ID Number: _____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

_____

(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are <u>not</u> allowed on this test. Calculators <u>are</u> necessary for certain parts of the test.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge and write your ID on both pages.
6. Number of questions = 7. Total Points = 75.

ID Number: _____

1. (10 points) A pool is in the shape of an inverted pyramid with a square base. The base has sides of length 10 ft and the pyramid has a height of 6 ft. The pool is full of water and the water is pumped out of a spout that extends 2 feet above the top of the pool. Set up but **do not evaluate** an integral that represents the amount of work needed to pump all of the water out of the pool. The density of water is 62.5 lbs/ft^3.

2. (10 points) Determine whether the sequence converges or diverges. If it converges, find the limit. Make sure to show your work!

(a) $a_n = \sqrt[n]{3^{1+2n}}$

(b) $a_n = \dfrac{e^{2n}+1}{e^n + e^{-n}}$

3. (10 points) Determine whether the series is convergent or divergent, and state which test you are using. **If it converges, find the exact sum. If you cannot find the exact sum, use a remainder estimate to find the sum to the nearest 0.01**.

(a) $\displaystyle\sum_{n=1}^{\infty} 2^{n+1} \cdot 10^{-2n+3}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-4)^n \cdot 7}{3^n(n+2)}$

4. (10 points) Determine whether the series is convergent or divergent, and state which test you are using. **If it converges, find the exact sum. If you cannot find the exact sum, use a remainder estimate to find the sum to the nearest 0.01**.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3+\cos n}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$

5. (10 points) Determine whether the series is convergent or divergent, make sure to state which test you are using.

(a) $\sum_{n=1}^{\infty} \pi^n \cdot e^{-n}$

(b) $\sum_{n=1}^{\infty} \frac{n^3 - 3n + 4}{3n^3 + n^2 + 1}$

6. (15 points) Find the radius of convergence and the interval of convergence of the series.

(a) $\displaystyle\sum_{n=3}^{\infty} \frac{(x-5)^n}{\sqrt[3]{n-2}}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n!\, x^n}{(n+1)8^n}$

7. (10 points) Determine whether the series is convergent or divergent, and state which test you are using. If it converges, state whether or not it is absolutely convergent.

$$\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n+1}}{n}$$

Extra Credit(2 points) For the series $\sum_{n=1}^{\infty} \frac{3}{n^2+7n+10}$, find the exact sum.